

03383168 MR 2003d#11163

**Nontrivial tame extensions over Hopf orders.**

Repleglo, Daniel R.

Underwood, Robert G.

(Underwood, Robert Gene)

Acta Arith.

Acta Arithmetica, 2002, 104, no. 1, 67--84. ISSN: 0065-1036

CODEN: AARIA9

Language: English

Document Type: Journal

Journal Announcement: 200215

Subfile: MR (Mathematical Reviews) AMS

Abstract Length: LONG (72 lines)

The classical Hilbert-Speiser theorem asserts that if  $L$  is a tame abelian extension of  $\mathbb{Bbb Q}$  with Galois group  $G$ , then the ring of integers  $\mathbb{O} \otimes L$  of  $L$  has trivial Galois module structure in the sense that  $L$  has a normal integral basis over  $\mathbb{Bbb Q}$ , or equivalently, that the class  $(\mathbb{O} \otimes L)$  of  $\mathbb{O} \otimes L$  in the locally free class group  $\mathrm{Cl}(\mathbb{Bbb Z}[G])$  is trivial. C. Greither et al. [J. Number Theory 79 (1999), no. 1, 164--173; MR 2000m:11111] showed that the analogous **statement** is **false** if  $\mathbb{Bbb Q}$  is replaced by any other number field  $K$ : there is a tame cyclic extension  $L/K$  of some prime degree  $l$  such that  $(\mathbb{O} \otimes L)$  is nontrivial in  $\mathrm{Cl}(\mathbb{K}[C \otimes 1])$ . The techniques used there are extended in the paper under review to certain wildly ramified extensions. In both cases, an important ingredient is L. R. McCulloh's determination [J. Algebra 82 (1983), no. 1, 102--134; MR 85d:11093] of the group  $\mathrm{scr R}(\mathbb{O} \otimes K[G]) = \{(\mathbb{O} \otimes L) \mid L/K \text{ tame, Galois with group } G\}$  of realisable classes in  $\mathrm{Cl}(\mathbb{O} \otimes K[G])$ , where  $G$  is an elementary abelian group of order  $l^{sp n}$ . His description of  $\mathrm{scr R}(\mathbb{O} \otimes K[G])$  is in terms of the action of a group  $C \cong C \otimes \{l^{sp n-1}\}$  of automorphisms of  $G$ .

To extend these ideas to wildly ramified extensions, one must first fix a Hopf order  $\Lambda \subset K[G]$  admitting the action of  $C$ . The reviewer [J. Algebra 177 (1995), no. 2, 409--433; MR 96h:11119] generalised McCulloh's result to the subgroup  $\mathrm{scr R}(\Lambda) \subset \mathrm{Cl}(\Lambda)$  of realisable classes over  $\Lambda$ , where in place of rings of integers  $\mathbb{O} \otimes L$  one must consider semilocal principal homogeneous spaces  $\mathrm{scr X}$  over the dual Hopf order  $\mathrm{scr B}$  of  $\Lambda$ . Thus  $\mathrm{scr X}$  is (without loss of generality) a subring of the ring of integers of some wildly ramified field extension  $L/K$  with Galois group  $G$ , and  $\mathrm{scr X}$  is a tame  $\Lambda$ -extension in the sense of L. N. Childs [Trans. Amer. Math. Soc. 304 (1987), no. 1, 111--140; MR 89a:11119] and coincides with  $\mathbb{O} \otimes L$  at places not above  $l$ . The key point of the paper under review is that  $\mathrm{scr R}(\Lambda)$  contains the classes of the generalised Swan modules  $r \Lambda + \{ \mathrm{scr L} \otimes \Lambda \otimes \Lambda \subset \Lambda$ , where  $\{ \mathrm{scr L} \otimes \Lambda \}$  is the ideal of integrals of  $\Lambda$  and  $r \in \mathbb{O} \otimes K$  is prime to  $\{ \mathrm{scr L} \} \otimes \Lambda$ . The group  $\mathrm{scr T}(\Lambda)$  of these Swan classes lies in the kernel group  $\mathrm{scr D}(\Lambda) = \ker(\mathrm{Cl}(\Lambda) \rightarrow \mathrm{Cl}(\Lambda))$ , where  $\mathrm{scr M}$  is the maximal order in  $K[G]$ . Moreover  $\mathrm{scr T}(\Lambda)$  surjects onto  $V \otimes \{ \mathrm{scr L} \otimes \Lambda \} \otimes \{l^{sp n-1}\}$ , where  $V \otimes \{ \mathrm{scr L} \otimes \Lambda \}$  is the quotient of  $\{ \mathrm{scr L} \otimes K \} \otimes \{ \mathrm{scr L} \otimes \Lambda \}$  by the image of  $\{ \mathrm{scr L} \otimes K \} \otimes \{l^{sp n-1}\}$ . Putting all this together the authors obtain their main result: If  $\Lambda$  is a Hopf order in  $K[G]$  admitting  $C$ , such that  $\{ \mathrm{scr L} \otimes \mathrm{scr B} \}$  is a principal ideal of  $\{ \mathrm{scr L} \otimes K \}$ , and if  $V \otimes \{ \mathrm{scr L} \otimes \Lambda \} \otimes \{l^{sp n-1}\} \neq 0$ , then  $\mathrm{scr R}(\Lambda) \cap \mathrm{scr D}(\Lambda) \neq 0$ .

To verify the condition on  $V \otimes \{ \mathrm{scr L} \otimes \Lambda \}$ , one requires a good knowledge of the unit group  $\mathbb{O} \otimes K \otimes \{l^{sp n-1}\}$ . In the last section of the paper,  $K$  is taken to be the cyclotomic field  $\mathbb{Bbb Q}(\zeta)$  of conductor  $l^{sp m}$ ,  $m \geq 1$ , where  $l > 3$  is a prime

satisfying Vandiver's conjecture. Thus the cyclotomic units in  $\mathbb{Q}(\zeta_p)$  have index prime to  $p$ . Let  $n=1$  or 2 and let  $\Lambda = \{\zeta_p\} \times K[(g-1)/(1-\zeta_p)] \cap \{g \in G\}$  with  $\zeta_p^{p-1} \in \{m-1\}$ . The authors show that their main result applies to  $\Lambda$ , so  $\Lambda \cap D(\Lambda) \neq 1$ . For  $n=1$ , the semilocal principal homogeneous spaces can be taken to be integrally closed over  $\mathbb{Bbb Z}$ , so that there exist rings of integers  $\mathbb{Q}$  locally free but not free over  $\Lambda$ . (The authors do not discuss the corresponding assertion for  $n=2$ , but in fact for  $\Lambda$  as above, no semilocal principal homogeneous space is integrally closed.)

Reviewer: Byott, Nigel (Exeter)

Review Type: Signed review

Descriptors: \*11R33 -Number theory-Algebraic number theory: global fields (For complex multiplication, see 11G15)-Integral representations related to algebraic numbers; Galois module structure of rings of integers (See also 20C10) ; 13C20 -Commutative rings and algebras-Theory of modules and ideals -Class groups (See also 11R29); 16W30 -Associative rings and algebras (For the commutative case, see 13-XX)-Rings and algebras with additional structure-Coalgebras, bialgebras, Hopf algebras (See also 16S40, 57T05); rings, modules, etc. on which these act

8/5/19 (Item 3 from file: 239)

DIALOG(R) File 239:Mathsci

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02157135 MR 90j#14064

**Smooth conjugacy of algebraic actions on affine varieties.**

Petrie, Ted (Department of Mathematics, Rutgers University, New Brunswick, New Jersey, 08903)

Corporate Source Codes: 1-RTG

Topology

Topology. An International Journal of Mathematics, 1988, 27, no. 4, 473-477. ISSN: 0040-9383 CODEN: TPLGAF

Language: English

Document Type: Journal

Journal Announcement: 2106

Subfile: MR (Mathematical Reviews) AMS

Abstract Length: LONG (29 lines)

From the text: ``Here is the main result: Theorem 1. The set of smooth conjugacy classes of algebraic actions of a compact Lie group on a nonsingular affine variety of real dimension at least 5 is countable. If algebraic is **replaced** by analytic, the **statement** is **false** .

``The motivation for this result is the linearity conjecture, which asserts that any algebraic action of a reductive algebraic group on affine  $\mathbb{R}^n$ -space is conjugate to a linear action. This topic has been popularized by H. Bass and H. Kraft (see, e.g., Kraft [in Geometry today (Rome, 1984), 251--265, Progress in Math., 60, Birkhauser Boston, Boston, MA, 1985; MR 88f:14042], Bass and W. Haboush [Trans. Amer. Math. Soc. 292 (1985), no. 2, 463--482; MR 87d:14039], Kraft, D. Luna and G. Schwarz [``Algebraic actions with 1-dimensional quotients'', Seminar talk, Rutgers Univ., New Brunswick, NJ; per bibl.] for the current status of the conjecture). In addition to being interesting, the conjecture is tough. It is not even known whether the set of algebraic conjugacy classes of algebraic actions of a reductive group of affine  $\mathbb{R}^n$ -space is countable, as implied by the linearity conjecture. Theorem 1 is a version of this countability question.

``We see in the course of the proof of Theorem 1 that there are an uncountable number of conjugacy classes of smooth actions of a nontrivial compact Lie group on real  $\mathbb{R}^n$ -space. Theorem 1 shows that only countably many of these are conjugate to algebraic actions on real  $\mathbb{R}^n$ -space.

``The main tools in the proof are the compactification theorem of Corollary 4 and the Palais rigidity theorem. Whether this rigidity theorem has an algebraic analogue is an interesting question whose consequences we briefly explore at the end.''

Reviewer: From the text

Review Type: Abstract

Descriptors: \*14L30 -Algebraic geometry-Group schemes (For linear algebraic groups, see 20Gxx. For Lie algebras, see 17B45)-Group actions on

✓ varieties or schemes (See also 14D25) ; 57R99 -Manifolds and cell complexes (For complex manifolds, see 32C10)-Differential topology (For foundational questions of differentiable manifolds, see 58Axx; for infinite-dimensional manifolds, see 58Bxx)-Topics not covered by other classifications in this subsection; 57S99 -Manifolds and cell complexes (For complex manifolds, see 32C10)-Topological transformation groups (See also 20F34, 22-XX, 54H15, 58D05)-Topics not covered by other classifications in this subsection

8/5/20 (Item 4 from file: 239)

DIALOG(R)File 239:Mathsci

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01712435 MR 83e#22010

**Sur les groupes  $\{\mathrm{Ext}\} \mathrm{sp}\{n\}$  des représentations des groupes de Lie résolubles.**

On the groups  $\{\mathrm{Ext}\} \mathrm{sp}\{n\}$  of representations of solvable Lie groups

Noncommutative harmonic analysis and Lie groups (Marseille, 1980)

Guichardet, A.

1981,

Springer, Berlin-New York,; pp. 179--196.,

Series: Lecture Notes in Math., 880,

Language: French Summary Language: English

Document Type: Proceedings Paper

Journal Announcement: 1411

Subfile: MR (Mathematical Reviews) AMS

Abstract Length: LONG (39 lines)

In this paper the author studies  $\mathrm{Ext}\{n\}$ -groups between irreducible unitary representations of certain solvable Lie groups. It would be desirable to describe these  $\mathrm{Ext}\{n\}$ -groups only in terms of parameters used for the Auslander - Kostant - Kirillov construction of irreducible unitary representations. A slightly more modest aim is to derive from these parameters some important properties of  $\mathrm{Ext}\{n\}$ -groups, for example triviality or reduction theorems. Now let  $(E_{\{sub\}j}, \mathrm{Pi}_{\{sub\}j})$  be irreducible unitary  $G$ -modules associated to  $(f_{\{sub\}j}, \chi_{\{sub\}j})$ ,  $f_{\{sub\}j} \in \mathrm{fraktr}g\{sup\ast\}$ ,  $\chi_{\{sub\}j} \in G(f_{\{sub\}j})\{sup\wedge\}$ , such that  $d \chi_{\{sub\}j} = f_{\{sub\}j} \{lgvert\} \{sub\} \{fraktr\}g(f_{\{sub\}j})$ , where  $j=1, 2$ . This means that we assume the  $f_{\{sub\}j}$  to be 'integral' in the sense of Auslander and Kostant. The author now formulates four desirable **statements** : (A) If  $G(f_{\{sub\}1})$  and  $G(f_{\{sub\}2})$  are not conjugate,  $\mathrm{Ext}\{n\}G(E_{\{sub\}1}, E_{\{sub\}2})$  is trivial for all  $n \geq 0$ . (B) If  $\Gamma := G(f_{\{sub\}1}) = G(f_{\{sub\}2})$  and  $\mathrm{Ext}\{n\}G(E_{\{sub\}1}, E_{\{sub\}2}) \neq 0$  for some  $n \geq 0$ , we have  $\mathrm{Ext}\{n\}G(E_{\{sub\}1}, E_{\{sub\}2}) \cong \mathrm{Ext}\{n\}\Gamma$ . (C) For a fixed  $E_{\{sub\}1}$  there are only a finite number of  $E_{\{sub\}2}$  such that  $\mathrm{Ext}\{n\}G(E_{\{sub\}1}, E_{\{sub\}2}) \neq 0$  for at least one  $n \geq 0$ . (D) If all roots of  $G$  are real and  $E_{\{sub\}1}$  and  $E_{\{sub\}2}$  are not equivalent,  $\mathrm{Ext}\{n\}G(E_{\{sub\}1}, E_{\{sub\}2}) = 0$  for all  $n \geq 0$ . The author then shows that in general, even in the simplest cases, the **statements** (A) - (D) are **false**. He therefore proposes to **substitute** the unitary modules by other modules, which still can be described by the Auslander - Kostant parameters, but have  $\mathrm{Ext}$ -groups, which can be computed more easily. One possibility is to take the differentiable  $G$ -modules which are associated to the unitary  $G$ -modules. In the nilpotent case this class of modules seems to be the right one, as many examples and the former results of the author suggest. Finally the author shows that for special semidirect products with abelian normal subgroup and suitably chosen  $G$ -modules  $E_{\{sub\}1}$  and  $E_{\{sub\}2}$  the **statements** (A), (C) and (D) hold and that (B) is correct for  $n=1$ . (For the entire collection see MR 82m:22001.)

Reviewer: Boidol, J. (Bielefeld)

Review Type: Signed review

Proceedings Reference: 82m#22001; 644 824

Descriptors: \*22E27 -Topological groups, Lie groups (For transformation groups, see 54H15, 57Sxx, 58-XX. For abstract harmonic analysis, see 43-XX) -Lie groups (For the topology of Lie groups and homogeneous spaces, see

57Sxx, 57Txx; for analysis thereon, see 43A80, 43A85, 43A90)-  
Representations of nilpotent and solvable Lie groups (special orbital  
integrals, non-type I representations, etc.)

8/5/21 (Item 5 from file: 239)

DIALOG(R) File 239:Mathsci

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01238379 MR 38##6639

**A certain property of a sequence of events.**

Taksar, M. I.

Teor. Verojatnost. i Primenen.

1968, 13, 531--534

Language: Russian Summary Language: English

Document Type: Journal

Subfile: MR (Mathematical Reviews) AMS

Abstract Length: SHORT (9 lines)

From the author's summary: ``Let  $\{A_n\}$  be a sequence of events and  $\lim \text{Pr}(A_n) = p > 0$ . Then it is possible to choose for any  $c < 1$  a subsequence with the property: for any  $k$ , the probability of the intersection of  $k$  arbitrary events of the subsequence is more than  $cp$ . This **statement** becomes **false** if we **replace** the constant  $c$  by a sequence  $c_k \rightarrow 1$ .''

\{This article has appeared in English translation [Theor. Probability Appl. 13 (1968), 501--504].\}

Reviewer: Thampuran, D. V.

Descriptors: \*60.05 -PROBABILITY THEORY AND STOCHASTIC PROCESSES-  
Probabilistic measure theory

8/5/22 (Item 6 from file: 239)

DIALOG(R) File 239:Mathsci

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01223681 MR 36##6713

**Behavior of solutions near a critical point.**

Proc. U.S.-Japan Seminar on Differential and Functional Equations  
(Minneapolis, Minn., 1967)

Sell, George R.

Sibuya, Yasutaka

1967,

Benjamin, New York; pp. 501--506,

Language: English

Document Type: Proceedings Paper

Subfile: MR (Mathematical Reviews) AMS

Abstract Length: MEDIUM (22 lines)

Let  $f(x, t)$  be bounded and uniformly continuous on sets of the form  $K \times R^1$ , where  $K$  is compact in  $R^n$ . Define the ``hull''  $H(f)$  of  $f$  as the closure of the set of translates  $f \circ \tau_{(x, t)} = f(x, t + \tau_x)$ . The authors investigate the behavior of the differential equation  $x' = f(x, t)$  at a critical point, imposing only the mild restriction that the differential equation  $x' = f \circ \tau_{(x, t)}$  admit unique solutions for all  $t \in H(f)$ . If the critical point is at  $x = 0$ , i.e., if  $f(0, t) \neq 0$ , their result is roughly the following: For every neighborhood  $U$  of zero, one of the **statements** below holds. (a) There exists a solution of  $x' = f(x, t)$  that tends to zero as  $t \rightarrow \infty$ . (b) For some  $t \in H(f)$  the equation  $x' = f \circ \tau_{(x, t)}$  has a solution that tends to zero as  $t \rightarrow -\infty$ . (c) For some  $t \in H(f)$  the equation  $x' = f \circ \tau_{(x, t)}$  has a solution that remains in  $U$  for all  $t$ . This considerably generalizes a result of T. Saito [Funkcial. Ekvac. 9 (1966), 199--206; MR 35#4523].

The **statement** becomes **false** if  $f \circ \tau_{(x, t)}$  is **replaced** by  $f$ . The proof leans heavily on previous work by the first author [Trans. Amer. Math. Soc. 127 (1967), 241--262; MR 35#3187a; ibid. 127 (1967), 263--283; MR 35#3187b].

Reviewer: Wasow, W.

Descriptors: \*34.50 -ORDINARY DIFFERENTIAL EQUATIONS-Asymptotic expansions, asymptotic behavior of solutions,

8/5/23 (Item 7 from file: 239)  
DIALOG(R) File 239:Mathsci  
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01200463 MR 34##341

**Polynomial automorphic forms and nondiscontinuous groups.**

Knopp, Marvin Isadore  
Trans. Amer. Math. Soc.  
1966, 123, 506--520

Language: English  
Document Type: Journal

Subfile: MR (Mathematical Reviews) AMS

Abstract Length: MEDIUM (25 lines)

Let  $\Gamma$  be a group of linear fractional transformations preserving the upper half-plane. It is easy to show that the existence of a nonconstant function  $f$  automorphic with respect to  $\Gamma$  implies that  $\Gamma$  is discontinuous. As the author shows, however, this **statement** becomes **false** if we **replace** 'automorphic function' by 'automorphic form'. That is, there are nondiscontinuous groups that support nonconstant automorphic forms. As might be expected, the groups and the functions on them that exhibit this behavior are very restricted in type.

The author shows that an automorphic form of even integral dimension  $r > 0$  with multipliers identically one on a nondiscontinuous  $\Gamma$  can only be a polynomial of degree  $\leq r$ . The possible polynomials are completely classified, as are the corresponding  $\Gamma$ . In certain cases the group on which a given polynomial is automorphic is discontinuous. For example, it is shown that when the polynomial (of degree at most  $r$ ) has at least three distinct zeros,  $\Gamma$  consists entirely of elliptic elements of order at most  $r+1$ ; thus  $\Gamma$  is finite (by a known theorem) and so discontinuous. The possible polynomials actually exist in each case; thus there are polynomial automorphic forms on both discontinuous and nondiscontinuous groups.

At the end of the paper the author considers multiplier systems  $\neq 1$  and arbitrary real dimension  $r$ .

Reviewer: Lehner, J.

Descriptors: \*30.49 -FUNCTIONS OF A COMPLEX VARIABLE-Automorphic functions and modular functions ; 10.22 -NUMBER THEORY-Automorphic forms, one variable

8/5/24 (Item 8 from file: 239)  
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01156344 MR 27##6252.

**A note on uncountably many disks.**

Martin, Joseph  
Pacific J. Math.  
1963, 13, 1331--1333

Language: English  
Document Type: Journal

Subfile: MR (Mathematical Reviews) AMS

Abstract Length: SHORT (10 lines)

An uncountable family of disjoint topological 2-spheres in 3-space always contains a tame 2-sphere [Bing, Trans. Amer. Math. Soc. 101 (1961), 294--305; MR 24#A1117]. Stallings has shown that if 'sphere' be **replaced** by 'disk', the above **statement** is **false** [Ann. of Math. (2) 71 (1960), 185--186; MR 22#1871].

In this note the author shows that under the Stallings hypotheses at least one disk of the family lies on a 2-sphere in 3-space. It appears to be still unsettled whether or not some pair of disks of such a family are equivalently embedded.

Reviewer: Harrold, O. G.

Descriptors: \*54.78 -GENERAL TOPOLOGY-Topology of  $E^{\infty}$ ,  $n$ -manifolds

(\$n\ge 2\$)

8/5/25 (Item 9 from file: 239)  
DIALOG(R) File 239:Mathsci  
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01149182 MR 26##6658

Properties of functions of several variables which can be sufficiently closely approximated by rational functions.

Dolzenko, E. P.

Izv. Akad. Nauk SSSR Ser. Mat.

1962, 26, 641--652

Language: Russian

Document Type: Journal

Subfile: MR (Mathematical Reviews) AMS

Abstract Length: MEDIUM (13 lines)

Continuing his investigations [Dokl. Akad. Nauk SSSR 130 (1960), 17--20; MR 22#9576], the author proves the following. Let  $E$  be a measurable set of the  $k$ -dimensional space,  $f(x)$  a bounded continuous function on  $E$  and  $R_{\sup n}(f, E)$  the deviation of  $f$  from rational functions  $R(x)$  of order  $\leq n$ , i.e.,  $R_{\sup n}(f, E) = \inf \sup \{x \in E\} \sqrt{f(x) - R(x)}$ . If  $\sum_{n=1}^{\infty} n^{p+1} R_{\sup n}(f, E) < \infty$  for a natural  $p$ , then the function  $f$  has almost everywhere on  $E$  a total differential of order  $p$ . The **statement** becomes **false** if 'total differential' is **replaced** by 'partial derivatives', except when  $p=1$  [see Goncar, ibid. 100 (1955), 205--208; MR 16, 803].

Reviewer: Lorentz, G. G.

Descriptors: \*41.17 -APPROXIMATIONS AND EXPANSIONS-Approximation by rational functions (See also 30.70)